MATH 1A - SOLUTION TO 4.4.14, 4.4.15, 4.4.49, AND 4.4.71

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1. 4.4.14

$$\begin{aligned}\lim_{\theta \to \frac{\pi}{2}} \frac{1-\sin(\theta)}{\csc(\theta)} &= \lim_{\theta \to \frac{\pi}{2}} \frac{-\cos(\theta)}{\frac{-\cos(\theta)}{\sin^2(\theta)}} \text{(l'Hopital's rule)} \\ &= \lim_{\theta \to \frac{\pi}{2}} \sin^2(\theta) \text{(cancelling out)} \\ &= 1\end{aligned}$$

2. 4.4.15

$$\begin{aligned}\lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} &= \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \text{by L'Hopital's rule} \\ &= \lim_{x \to \infty} \frac{2\sqrt{x}}{x} \text{invert and multiply rule for fractions} \\ &= \lim_{x \to \infty} \frac{2}{\sqrt{x}} \\ &= 0\end{aligned}$$

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3. 4.4.49

$$
\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2+x} - x &= \lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right) \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} \text{(use conjugate form)} \\
&= \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to\infty} \frac{x}{\sqrt{x^2}\cdot\sqrt{1+\frac{1}{x}}+x} \text{(factor out } x^2 \text{ out of the square root)} \\
&= \lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x}}+x} (\sqrt{x^2} = |x| = x, \text{ since } x > 0) \\
&= \lim_{x\to\infty} \frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} \\
&= \frac{1}{\sqrt{1}+1} \\
&= \frac{1}{2}
\end{aligned}
$$

4. 4.4.71

4.1. **a.** Here, you want to calculate $\lim_{t\to\infty}$, so **treat** t **as our** x**, and leave everything else as a constant!!!!**. In particular, we get:

$$\lim_{t\to\infty} e^{-\frac{ct}{m}} = 0 \qquad \text{because } c > 0 \text{ and } m > 0$$

So, we get:

$$\lim_{t\to\infty} v = \frac{mg}{c}(1-0) = \frac{mg}{c}$$

4.2. **b.** Here, we let $c \to 0^+$, so **we treat c as our x, and leave everything else as a constant!**.

Then, we have:

$$\begin{aligned}
\lim_{c\to 0^+} v &= \lim_{c\to 0^+} mg\left(\frac{1-e^{-\frac{ct}{m}}}{c}\right)\\
&= \lim_{c\to 0^+} mg\left(\frac{-\left(-\frac{t}{m}\right)e^{-\frac{ct}{m}}}{1}\right) \qquad \text{(by l'Hopital's rule)}\\
&= mg\left(\frac{\left(\frac{t}{m}\right)e^{0}}{1}\right)\\
&= mg\left(\frac{\left(\frac{t}{m}\right)1}{1}\right)\\
&= \frac{mgt}{m}\\
&= gt
\end{aligned}$$